FLUID MECHANICS -ME

MECHANICAL ENGINEERING

MECHANICAL ENGINEERING-ME

GATE / PSUs

STUDY MATERIAL

FLUID MECHANICS-FM

MECHANICAL ENGINEERING

GATE & PSUs

STUDY MATERIAL

FLUID MECHANICS

CONTENT

CHAPTER-1

PROPERTIES OF FLUID

1.1 Fluid: Liquids and gasses both are having the property of continuous deformation under the action of shear or tangential forces, this property of continuous deformation is known as flow property, where as this property is not found in solids, hence liquids and gasses both are kept in different category which is for away from solids and this category is known as 'fluids.'

"A fluid is a substance that deforms continuously under the action of a shear (tangential) stress no matter how small the shear stress may be."

Example: (as shown in figure), if a shear stress is applied at any location in a fluid, the element oxx' which is initially at rest, will move to oyy', then to ozz' and so on.

→ The tangential stress in a fluid body depends on the velocity of deformation, and vanishes as the velocity approaches zero.

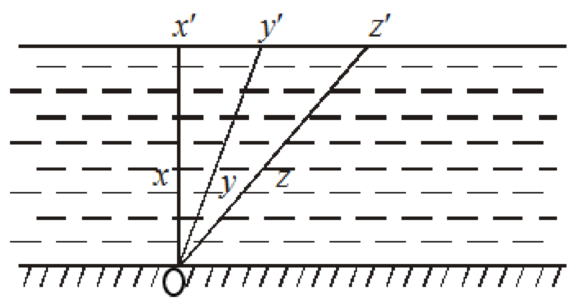


fig : shear stress on a fluid only

1.2 Fluid is Continuum

In macroscopic system of fluid particles the inter molecular distances can be treated as negligible as compare to the characteristics dimension of systems, so therefore we can assume adjacent to one molecule there is another molecule and there is no inter space between them so the entire fluid mass system can be treated as a continuous distribution of mass which is called continuum.

1.3 Properties of Fluid:

$$1 \text{ dyne} = 1\,\text{gram} \times \frac{1\,\text{cm}}{\text{sec}^2}$$

$$1 \text{ kilogram} = 1\,\text{metric slug} \times \frac{1\,\text{m}}{\text{sec}^2}$$

$$1\text{ pound} = 1\,\text{pound} \times \frac{1\,\text{ft}}{\text{sec}^2}$$

$$1\text{ pound} = 1\,\text{slug} \times \frac{1\,\text{ft}}{\text{sec}^2}$$

(i) Density/mass density, ρ: $\rho = \frac{M}{V}$

- → The density of liquids may be considered as constant while that of gasses changes with the variation of pressure and temperature.
- → Atmospheric air $\rightarrow 1.21\ \text{kg/m}^3$

  Density of H_2O $\rightarrow$ 1000 kg/m^3

Where, ρ: density (kg/m^3)

M: mass (kg)

V: volume (m^3)

→ ρ_w(water) = 1000 kg/m^3 or 1g/cm^3

- 'ρ' Depends on temperature and Pressure. Temperature $\uparrow \Rightarrow \rho \downarrow$
  Pressure $\uparrow \Rightarrow \rho \uparrow$

(ii.) Specific Weight or Weight Density

As it stands for the force exerted by gravity on a unit volume of a fluid, it has units force per unit volume.

→ SI unit – N/m^3

→ Weight density depends on the gravitational acceleration and mass density. Since the gravitational acceleration (g) varies from place to place, the specific weight will also vary.

→ The mass density changes with temperature and pressure, hence the specific weight will also depend upon temperature and pressure.

$$\omega = \gamma = \frac{\text{weight}}{\text{volume}} = \frac{\text{mg}}{\text{V}}$$

$$\omega = \gamma = \rho g$$

Where, ω = γ: specific weight. (N/m^3)

m: mass (kg)

g: acceleration due to gravity = 9.81 m/s^2

V: volume (m^3)

→ **γ_w(water) :** 9.81 kN/m^3

(iii) Specific volume *v*: $= \frac{V}{w} = \frac{1}{\rho g} = \frac{1}{\gamma} = \frac{1}{\omega}$ Defined as volume per unit weight of a fluid.

→ Thus it is reciprocal of specific weight

→ SI unit – m^3/N

→ Metric gravitational system unit – $m^3/kg(f)$

→ In the metric absolute system – $cm^3/dyne$ or cc/dyne

→ In the English gravitational and absolute unit *i.e.* $ft^3/slug$

→ For the problems involving the gas flow specific volume is defined as the volume of the fluid per unit mass, in which it's reciprocal of mass density.

→ SI units – m^3/kg

→ For liquids the ρ, γ, υ vary only slightly with the variation of pressure and temperature.

It is because of molecular structure of liquids in which the molecules are arranged compactly, (compared to gas)

→ For gasses the values of ρ, γ, υ properties vary greatly with variation of either pressure, or temperature or both. It is because to the molecular structure of the gas in which the molecular spacing (*i.e.* volume) changes considerably due to the pressure and temperature variations.

(*iv*) **Specific gravity (G):** Defined as the ratio of specific weight or mass density of a fluid to the specific weight or mass density of a standard fluid.

→ For liquid, **water** is taken as standard fluid.

→ For gas, **air** is taken as standard fluid.

→ For liquid, $G = \frac{\gamma_1}{\gamma_w}$

Where,

γ_l = specific weight of liquid

γ_w = specific weight of water

→ For gas, $G = \frac{\gamma_g}{\gamma_{air}}$

γ_g = specific weight of gas

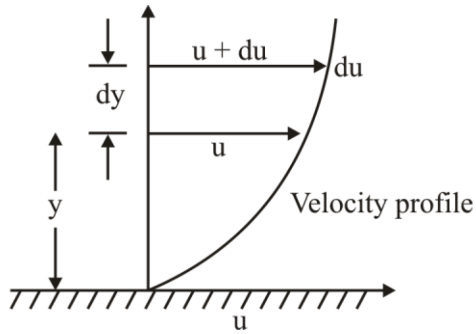
γ_{air} = specific weight of air

→ For, mercury, G = 13.6

→ For liquids, the standard fluid chosen for comparison is pure H_2O at 4ºC.

→ For gasses H_2 or air at some specified temperature and pressure.

1.4 Viscosity: When a layer of fluid resist the motion of an adjacent layer such a fundamental property of fluid is called 'viscosity'.



Hence, Newton's equation of viscosity. Mathematically,

$$\tau \propto \frac{du}{dy}$$

$$\Rightarrow \qquad \boxed{\tau = \mu \frac{du}{dy}}$$

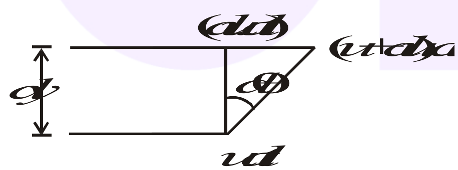
Where, τ = shear stress (N/m^2)

$\frac{du}{dy}$ = rate of shear strain (1/s) OR rate of shear deformation OR velocity gradient

μ= Co-efficient of dynamic viscosity or viscosity or viscosity of fluid

$\rightarrow$ The relative viscosity of layer which is in contact with the surface is zero.

$\rightarrow$ There is the development of velocity gradient in transverse direction of flow $\left(\frac{du}{dy}\right)$



Angular shear deformation = $\tan\theta = \frac{du\,.\,dt}{dy}$

$$\frac{d\theta}{dt} = \frac{du}{dy} \quad \text{(If } \theta \text{ is very small)}$$

Rate of angular deformation = Velocity gradient in transverse direction of flow.

Unit of μ :

In SI unit, $\mu = N.s/m^2$ or Pa.s $\left[\because 1\,N/m^2 = 1\,Pa\right]$

In CGS unit, $\mu = \frac{\text{dyne} - \text{sec}}{cm^2} = 1\,\text{poise}$

In MKS unit, $\mu = \frac{\text{kgf} - \text{sec}}{m^2}$

$\rightarrow\ 1\dfrac{\text{N}-\text{S}}{\text{m}^2} = 10\,\text{poise}$

$\rightarrow$ 1 Centi-poise (=1 cp) = $\dfrac{1}{100}\text{poise}(=\text{p})$

$\rightarrow$ Viscosity of water (μ) at 20ºC = 0.01 poise or 1 cp.

$$\mu = \frac{\tau}{\frac{d\theta}{dt}}$$

$\rightarrow$ If μ is high $\rightarrow \dfrac{d\theta}{dt}$ is less

$\rightarrow$ flow is difficult

$\rightarrow$ If μ is less $\rightarrow \dfrac{d\theta}{dt}$ is high

$\rightarrow$ flow is easy

$\rightarrow$ It means that viscosity is direct measurement of the internal resistance between the two layers in flow.

1.5 Kinematic viscosity, υ: Defined as the ratio of co-efficient of dynamic viscosity (μ) to the density (ρ) of fluid. $\boxed{\upsilon = \dfrac{\mu}{\rho}}$

Unit of υ :

$\rightarrow$ In SI unit, υ m^2/s

$\rightarrow$ In CGS unit, υ cm^2/sec = stoke

$\rightarrow$ 1 Stoke = 10^{-4} m^2/s and 1 Centistokes = 1/100 stoke

- **Variation of viscosity with temperature**:

For Liquid: $\boxed{\mu = \mu_0\left\{\dfrac{1}{1+\alpha t+\beta t^2}\right\}}$

μ : Viscosity of liquid at tºC (in poise)

μ_0 : Viscosity of liquid at 0ºC (in poise)

α, β: Constants

$\rightarrow$ With increase in temperature, viscosity decreases.

$\rightarrow$ Here, cohesive forces predominate, which get reduced with increase in temperature.

For Gas: $\boxed{\mu = \mu_0 + \alpha t - \beta t^2}$

→ With increase in temperature, viscosity increases.

→ Here, molecular momentum transfer predominates which increases with increase in temperature.

1.6 Types of fluids:

(i) Ideal fluid:

→ Non-viscous, incompressible

→ Surface tension doesn't exist

→ Offers no resistance against flow

→ Also known as **imaginary fluid**

→ For mathematical analysis, fluids with low viscosity are treated as ideal fluid.

Example:- Air, water etc.

(ii) Real fluid:

→ Viscous, compressible

→ Surface tension exists

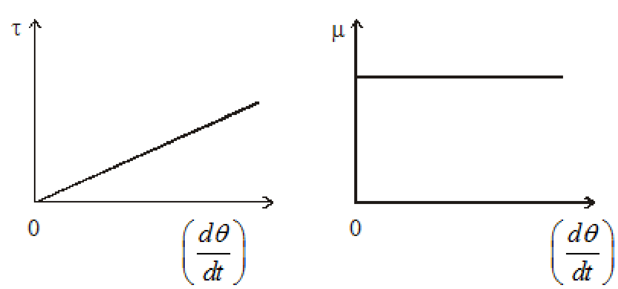
→ Offers resistance against fluid

→ Mostly fluid available in nature are real fluid

1.7 Newtonian fluid: The flow which obey Newton law of viscosity *e.g.* water, oil, air.

$$\boxed{\tau = \mu \frac{du}{dy}}$$

→ $\boxed{\tau = \mu \dfrac{d\theta}{dl} = \mu \dfrac{du}{dy}}$ $(\mu = \text{constant})$



Example: air, water, Glycerin, kerosene etc.

1.8 Non-Newtonian Fluid: These don't obey Newton's law of viscosity i.e. $\boxed{\tau \neq \mu \frac{du}{dy}}$

→ **Ideal plastic:** It has a definite yield stress and a linear relation exists between shear stress (τ) and rate of shear strain $\frac{du}{dy}$.

→ **Thixotropic fluid:** It has a definite yield stress and a non-linear relation exists between shear stress (τ) and rate of shear strain (du/dy) Example: Printer's ink

→ As $T\uparrow \Rightarrow \mu\downarrow$

1.9 Bulk modulus, K: Defined as the ratio of compressive stress to volumetric strain.

$$\boxed{K = \frac{dP}{\left(\frac{dV}{dV}\right)}}$$

Where, dP : change in pressure = compressive stress.

$\frac{dV}{V}$: Volumetric strain = $\frac{\text{change in volume}}{\text{original volume}}$

→ K (for water) at normal temperature and pressure = 2.06×10^9 N/m^2

→ K (for air) at normal temperature and pressure = 1.03×10^5 N/m^2

→ Hence, air is about 20,000 times more compressible than water.

→ It is temperature dependent.

1.10 Compressibility: It is given as the reciprocal of bulk modulus (K).

$$\beta = \frac{\left(\frac{dV}{V}\right)}{dP} = \frac{1}{K}$$

$$\text{mass} = m = \rho V = \text{constant}$$

$$\rho dV + V d\rho = 0$$

$$-\frac{dV}{V} = \frac{d\rho}{\rho}$$

$$\beta = \frac{1}{\rho} \cdot \frac{d\rho}{dP}$$

- If ρ is not changing with respect to pressure

$$\frac{d\rho}{dP} = 0 \quad \Rightarrow \quad \boxed{\beta = 0}$$

Fluid is incompressible

- If ρ is changing with respect to pressure

$$\frac{d\rho}{dP} \neq 0 \quad \Rightarrow \quad \boxed{\beta \neq 0} \text{ Fluid is compressible}$$

- **Liquid**

$$\begin{array}{ccc} \text{Compressible} & \text{Pressure} & \text{Density} \\ H_2O & 1\,\text{atm} = & 998\,\text{kg/m}^3 \\ \rightarrow & 100\,\text{atm} = & 1003\,\text{kg/m}^3 \\ \hline & \Delta\rho = 5\,\text{kg/m}^3 & \end{array}$$

$$\%\text{ change} \Rightarrow \left(\frac{5}{998} \times 100\right) = 0.5\%$$

very less $\simeq 0$

Therefore, liquid are treated as incompressible

- **Gas:** Highly compressible

  Mach number $(M_a) = \dfrac{V_{object}}{V_{sound}}$

  [If mach number ≤ 0.3 gas are treated as incompressible]

- The reciprocal of compressibility is called 'Bulk modulus of elasticity'

1.11 Surface tension:

Fluids are having very important property by virtue of which it is minimize its surface area upto its certain limit, such a property of fluids is known as 'surface tension'

$\rightarrow$ Basic cause of surface tension is cohesion

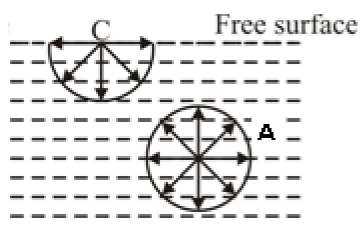
$\rightarrow$ Mathematically

$$\sigma = \frac{F}{L} \text{ unit N/m}$$

Where, σ : surface tension (N/m)

T : tensile force (N)

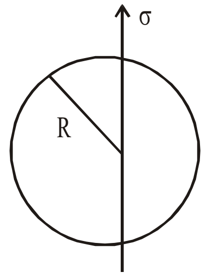
L : Length (m)



- It is temperature dependent and decreases with rise in temperature.
- It is also dependent on the fluid in contact with the liquid surface.

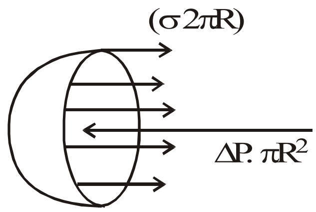
Case I: Pressure Intensity inside a liquid a droplet:

P_i = inner pressure



P_o = outer pressure

$\Delta P = P_i - P_o$ = excess pressure



A_c = circumferential area

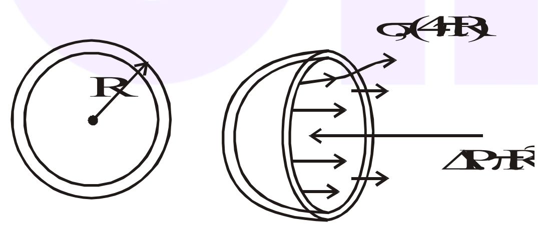
$\Delta P \, . \, \pi R^2 = \sigma \, . \, 2\pi R$ [Surface tension × circumferential area]

$$\boxed{\Delta P = \frac{2\sigma}{R}}$$

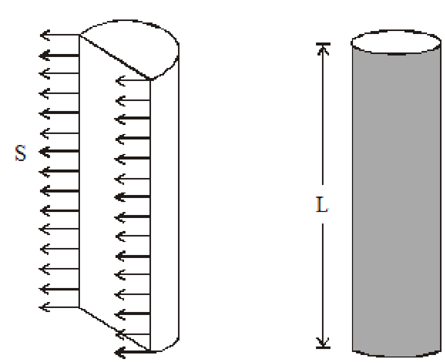
Case II – Pressure Intensity inside a soap bubble:

$\Delta P \pi R^2 = 4\sigma\pi R$

$$\boxed{\Delta P = \frac{4\sigma}{R}}$$



Case III: Surface tension on a liquid jet considers a jet of length *l*, and diameter *d*.



Force due to pressure = P × L × *d*

Force due to surface tension = $\sigma \times 2L$

At equilibrium $P \times L \times d = \sigma \times 2L$ $\boxed{P = \frac{\sigma \times 2L}{L \times d} = \frac{2\sigma}{d}}$

Where, P – Pressure intensity inside the jet (N/m^2)

σ – Surface tension (N/m)

L – Length of jet (Assumed)

d – Diameter of Jet

1.12 Wetting and Non-wetting liquids:

$\rightarrow$ It depends on both cohesion and adhesion

$\rightarrow$ It is mutual property of liquid-surface

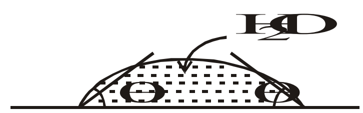
If Adhesion >>>> cohesion

(wetting) (water – glass) $\left(\theta < \frac{\pi}{2}\right)$

$128^o - 130^o$

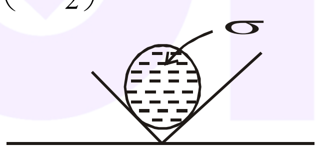
θ For Hg $128^o - 130^o$

θ For water 0^o



If cohesion >>>> Adhesion

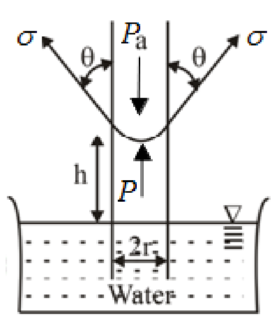
(Non-wetting) (Hg-glass) $\left(\theta > \frac{\pi}{2}\right)$



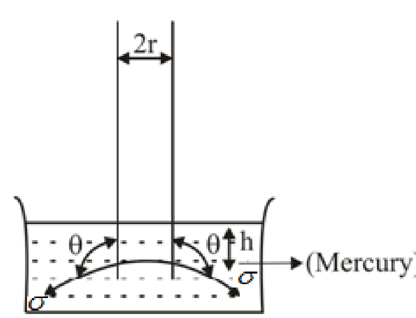
$[\theta = 130^o - 140^o]$

1.13 Capillarity:

When a tube of very fine diameter is immersed in liquid then there may be the rise or fall of liquid level inside the tube depending upon the wetting and non-wetting behavior of the liquid with the tube surface, this rise or fall of liquid level is a phenomenon known as ‘capillarity’ and the tube is called ‘capillarity tube’.



Capillary rise



Capillary depression

When $P>P_a$ When $P < \rho_a$

- Capillary, rise or fall $h = \dfrac{2\sigma\cos\theta}{r\rho g}$

Where, h : capillary rise or fall (m).

θ : Angle of contact between liquid and glass tube.

ρ : density of liquid (kg/m^3)

g : 9.81 m/s^2 (Acceleration due to gravity)

σ : surface tension (N/m)

- For water and clean glass tube, $\theta = 0^o$
- For mercury and glass tube, $\theta = 128^o$
- Above equation of capillary rise or fall holds true for small radius, r < 2.5 mm.
- For radius of tube $r \geq 6$ mm, value of 'h' becomes negligible.

Special case:

If the tube of radius, r is inserted in a mercury of specific gravity (S_1) and above which another liquid of specific gravity (S_2) lies then capillary rise or fall $\boxed{h = \dfrac{2\sigma\cos\theta}{r\gamma_w(S_1 - S_2)}}$

γ_w : specific weight of water.

1.14 **Vapour Pressure:**

- All liquids posses a tendency to evaporate or vaporize, that is, to change from the liquid to gaseous state.
- i.e vaporization occurs due to continuous escaping of the molecules through the free liquid surface.
- If the liquid is confined in a closed vessel, the ejected vapour molecules get accumulated in the space between the free liquid surface and fop of the vessel. This accumulated vapour of the liquid exerts a partial pressure on the liquid surface which is called as vapour pressure of the liquid.
- When the molecular activity increases with temperature, vapour pressure of the liquid also increases with temperature.

1.15 Cavitation: When for a flowing fluid the local static pressure <= vapour pressure of the fluid then the vapoursation of the fluid starts and this vapour accumulated in the low pressure region where babble formation starts. Which erodes the pipe material which is called Cavitation

Question and Solution

Example-1: At a certain point in castor oil the shear stress is 0.216 N/m^2 and velocity gradient 0.216 $\sec^{-1}$. If the mass density of castor oil is 959.42 kg/m^3 find kinematic viscosity

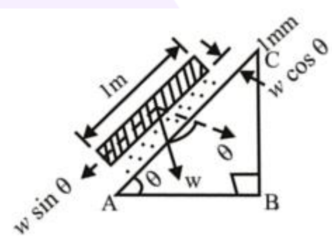
Solution: Kinematic viscosity $\upsilon = \dfrac{\text{Dynamatic Viscosity}(\mu)}{\text{Density }(\rho)}$

$$\tau = \mu\left(\frac{du}{dy}\right) \qquad 0.216 = \mu(0.216) \qquad \mu = 1 \text{ N.S/m}^2$$

$$\upsilon = \frac{1}{959.42} = 1.042 \times 10^{-3} m^2 / \sec$$

Example 2: A square plate of size 1m × 1m and weighting 350 N slides down an inclined plane with a uniform velocity of 1.5 m/sec. The inclined plane is laid on a slope of 5 vertical to 12 horizontal. Oil film thickness = 1mm. Calculate the dynamic viscosity.

$\mu = ?$



Solution: $F = W \sin\theta = 350 \times \dfrac{5}{13} = 134.615$

$$\tau = \mu\left(\frac{du}{dy}\right) \qquad du = u - 0 = 1.5 \text{ m/sec}$$

$$\frac{F}{A} = \mu\frac{du}{dy} \qquad \mu = \left(\frac{134.615}{1}\right) \times \frac{1\times10^{-3}}{1.5}$$

$$\boxed{\mu = 0.897 \text{ Poise}}$$

$$\therefore \tan\theta = \frac{5}{12} \qquad \therefore Ac = \sqrt{AB^2 + BC^2} = 13$$

$$\sin\theta = \frac{5}{13} \qquad \therefore dy = 1 \times 110^{-3} m$$